Two-phase isotropic composites with prescribed bulk and shear moduli.

T. Łukasiak
Warsaw University of Technology, Warsaw, Poland.

ABSTRACT: The paper deals with the inverse homogenization problem: to reconstruct the layout of two elastic and isotropic materials given by bulk ($\kappa_2 > \kappa_1$) and shear ($\mu_2 > \mu_1$) moduli within a hexagonal (2D) periodicity cell, corresponding to the predefined values of the bulk and shear moduli (κ^*, μ^*), of the effective isotropic composite and to the given isoperimetric condition concerning the volume fractions. The effective isotropic moduli are computed according to the homogenization algorithm, with using appropriate Finite Elements (FE) techniques along with periodicity assumptions. The inverse problem thus formulated can be effectively solved numerically by the Sequential Linear Programming (SLP) method. The isotropy conditions, usually explicitly introduced into the inverse homogenization formulation, do not appear in the algorithm, as being fulfilled by the microstructure construction. The rotational symmetry of angle 120° of the resulting representative volume element is assumed.

1 INTRODUCTION

The topology optimization problems which state the questions on the optimal layout of several materials within a feasible domain Ω of dimension D assume a correct mathematical form only if they are *a priori* relaxed, see Allaire (2002). Relaxation means admitting infinitely dense mixtures of the constituents, which is equivalent to adopting the hierarchical composites to the design problem discussed. For instance, if in the initial formulation two materials C_1 and C_2 (of Hooke tensors $\mathbf{C}_1$, $\mathbf{C}_2$ or equivalent constitutive matrices $\mathbf{E}_1$, $\mathbf{E}_2$, respectively) are at our disposal to fill up the domain Ω, the relaxed formulation should admit three materials: C_1, C_2 and C_ρ, the latter being a composite of Hooke's moduli tensor $\mathbf{C}_\rho(\rho_2)$ formed by mixing the materials C_1 and C_2 in proportions ρ_1 and $\rho_2 = 1 - \rho_1$. The moduli of the composite are compatible with the theory of homogenization of periodic composites (Sanchez-Palencia 1980), the periodicity cell Y being filled up with the given materials C_1 and C_2, the periodicity cell Y replaces the representative volume element (RVE), which is not any restriction (Torquato 2002). The best known problem of topology optimization concerns minimization of the compliance of the two material structure, built from two isotropic materials of given volumes. The relaxed problem assumes the form referring to a composite: the unknown is the density ρ_2 at each point of the feasible domain Ω. Upon solving the relaxed problem we obtain the energy density of the material and the densities $\rho_1(\mathbf{x})$, $\rho_2(\mathbf{x})$ at each point $\mathbf{x}$ of Ω. Thus the distributions of the latter functions are achievable by relaxation. However, this determinacy ceases to hold at the microlevel. One cannot expect that the solutions to the relaxed problems of topology optimization would determine the microproperties. We can indicate various microstructures realizing the same relaxed solution, including the density of energy at each point of Ω. In the minimum compliance problem there is a theorem at our disposal which says that the solution can be attained by orthogonal laminated microstructures of 2nd (D =2) or 3rd rank(D = 3), see Allaire (2002). In particular, if D = 2, the domain Ω is divided into 3 subdomains (I, II, III). In domain II the minimum of the compliance is attained on 1st rank microstructures which smoothly change into 2nd rank microstructures along the boundaries between the I and II, and II and III subdomains. This kind of solution is possible but is not the only one. It is known that in the regions II one can indicate other microstructures than laminates giving exactly the same energy density and refer to the same ρ_1, ρ_2 values. One can say only for sure that in the domains I and III neither 1st rank microstructure suffices. Thus this example shows that the relaxed formulations of topology optimization problems do not de

unit cells) are shown in Figures 12 and 15.

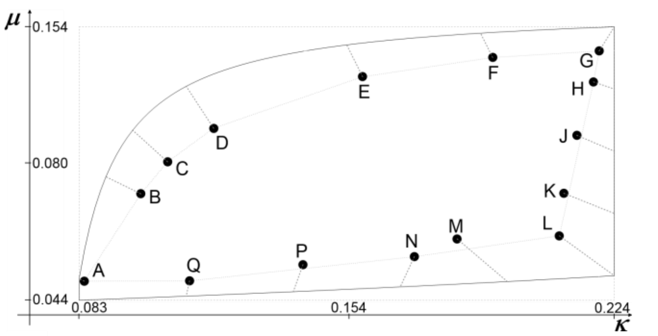

Figure 10. Selected points (κ, μ) within the C-G domain for *S1*.

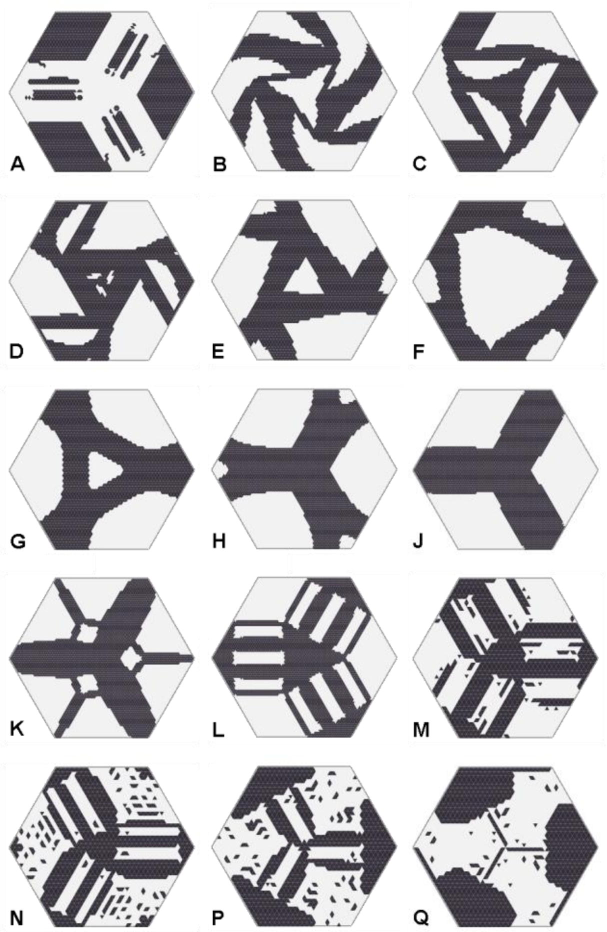

Figure 11. The reconstructed 01 periodic cells for *S1*.

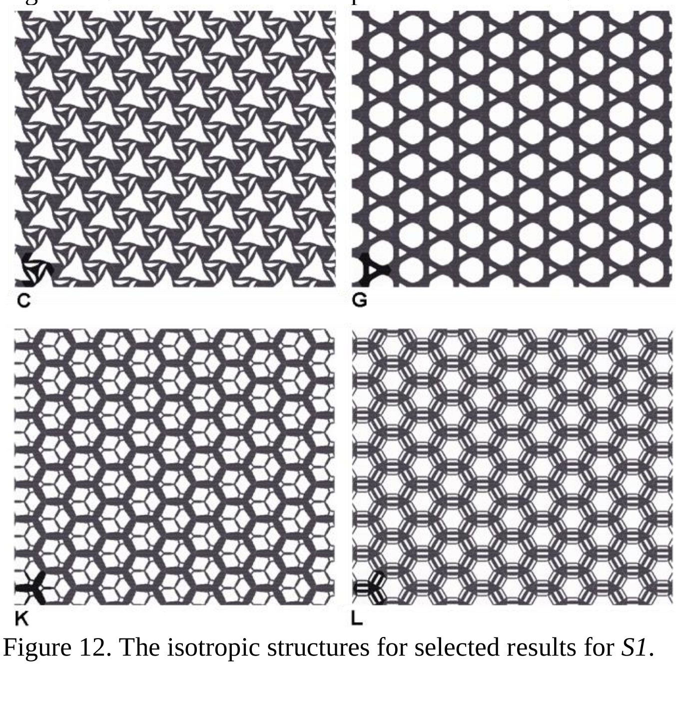

Figure 12. The isotropic structures for selected results for *S1*.

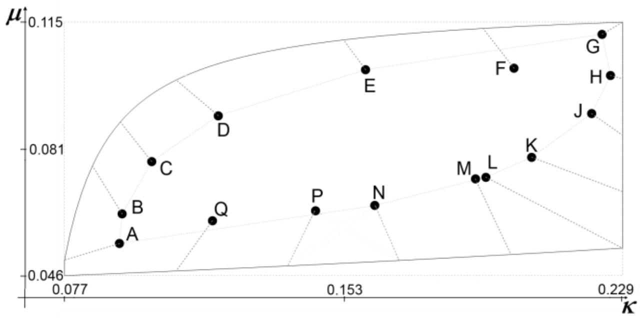

Figure 13. Selected points (κ, μ) within the C-G domain for *S2*

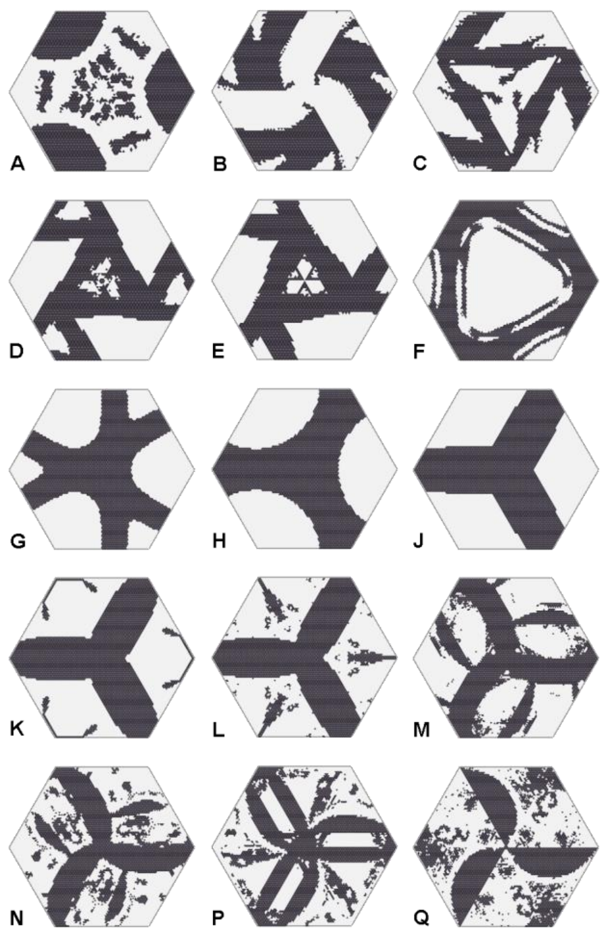

Figure 14. The reconstructed 01 periodic cells for *S2*.

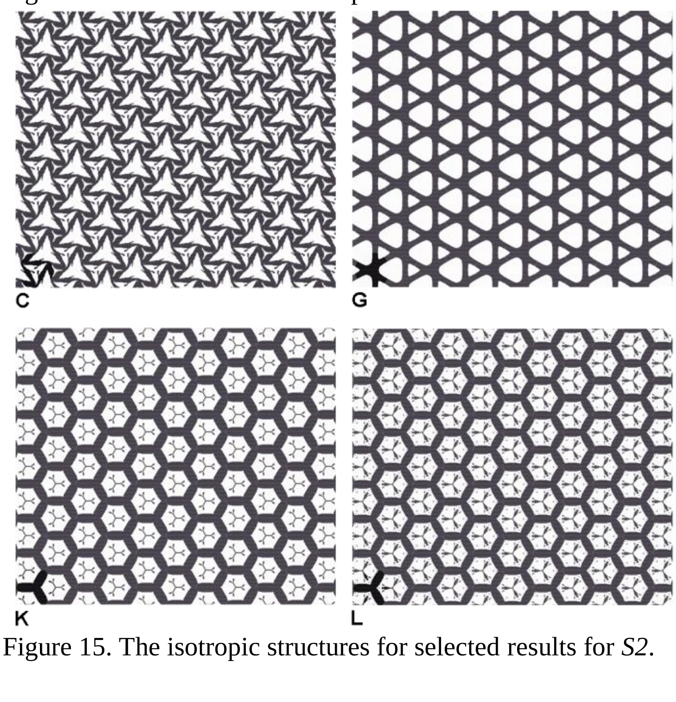

Figure 15. The isotropic structures for selected results for *S2*.

The results obtained for materials with the ν ratio are much better than the results obtained for materials with different ratio ν (compare Figures 10 and 13). For the same value of ν adopted for both materials (for *S1)* the obtained points (κ, μ) are lying much closer to the assumed points (κ^*, μ^*) that in case of different values of ν (for *S2*). Influence of adopted coefficients is particularly evident for the cases K, L, M, N, P, Q. It should be noted that a similar effect occurs in cases for when the ρ_2 is close to 0 or 1. This is probably the effect of the method used to obtain the 0-1 solution (d = 1 in the last step of the SLP). It is necessary to emphasize the fact, that close to each other structures in terms of (κ, μ) can be significantly different in the topology (compare e.g. K and L cases for *S1* or M and L for *S2*).

2 FINAL REMARKS

The present paper puts forward a new algorithm of the numerical inverse homogenization for the planar isotropic composites. The paper is aimed at finding the rank-1 subclass of the isotropic composites of effective moduli achieving the C-G bounds. The key point of this algorithm is the use of the hexagonal cell of periodicity instead of a rectangular cell. It is assumed that the internal structure of the hexagonal cell possesses rotational symmetry of angle 120 degree. This assumption results in a significant reduction in the number of design variables to the optimization problem considered (approximately 6 times less than for the case of the cell of a rectangular shape). Moreover, such a cell shape ensures isotropy of any periodic composites of such class thus essentially reducing the number of constraints involved in the optimization problem. The optimization problem considered has been solved by the SLP method augmented with appropriate filters. It is worth emphasizing that the results obtained lie fairly close to the assumed ones located on the C-G bound, but the bounds are not attained. The presented results show that the points along the C-G bound are attainable only within the class of microstructures of the rank higher than one.

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